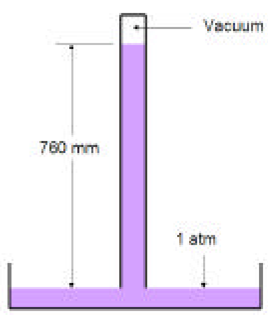
Homework #1 Due: Sept. 3

Name ________________________ Chemistry 331

1. A mercury barometer is shown in the figure. One atmosphere of pressure corresponds to a supported column of Hg that is 760 mm high.
   A. Calculate the density of mercury. Show your work.
   B. Suppose the barometer is taken to the top of Mt. Mitchell, the highest point in North Carolina. Assume that Mt. Mitchell is 2000 m in elevation at the top. How high will the column of Hg be at that elevation.



Solution:

A. $P = \rho g h \rightarrow \rho = P/gh = (1.01325 \times 10^5 \text{ Pa})/(9.8 \text{ m/s}^2)/0.76 \text{ m})$
$= 13604 \text{ kg/m}^3 = 13.6 \text{ g/cm}^3$.

B. Calculate the pressure at the top of Mt. Mitchell using the barometric pressure formula.

$$P = \exp\left\{-\frac{Mgh}{RT}\right\} = \exp\left\{-\frac{(0.029\ kg/mol)(9.8\ m/s^2)(2000\ m)}{(8.31\ J/mol\text{–}K)(298\ K)}\right\} = 0.795\ atm$$

For this part we can accept M = 0.028 kg/mol if you just used the value for N_2.
We can also accept g = 10 m/s^2 as an approximate value.
If the atmospheric pressure is reduced to 0.795 atm the column of Hg will be reduced by the same factor so it will be:

$$h_2 = h_1\left(\frac{P_2}{P_1}\right) = 760\ mm\left(\frac{0.795\ atm}{1\ atm}\right) = 604\ mm$$

2. What is the density of air inside a bicycle tire that has been inflated to 45 lbs./in.^2 of pressure?

Solution: First we convert to sane units. 1 lb. = 454 grams, 1in. = 2.54 cm = 0.0254 m
$P = 45(0.454 \text{ kg})(9.8 \text{ m/s}^2)/(0.0254 \text{ m})^2 = 3.1 \times 10^5 \text{ Pa} \sim 3 \text{ atm}$.
As a general rule it is good to remember that 15 lbs./in^2 = 1 atm (until we come to our senses and go metric).
The density of a gas is
$\rho = MP/RT = (0.029 \text{ kg/mol})(3.1 \times 10^5 \text{ Pa})/(8.31 \text{ J/mol-K})/(298 \text{ K}) = 3.63 \text{ kg/m}^3 = 0.0036 \text{ g/cm}^3$.

3. A child releases a He balloon into the air. Assuming that the balloon is inelastic (has a constant shape) determine the elevation to which it will rise.

   Solution: The balloon will rise until the density of the air is equal to the density of the gas inside the balloon (He gas at 1 atm). First we calculate the density of the gas inside the balloon.
   $\rho = MP/RT = (0.004\ kg/mol)(10^5\ Pa)/(8.31\ J/mol\text{-}K)/(298\ K) = 0.161\ kg/m^3 = 1.61 \times 10^{-4}\ g/cm^3$.
   We will also need to know the density of the atmosphere at sea level as a reference.
   $\rho_0 = M_0P/RT = (0.029\ kg/mol)(10^5\ Pa)/(8.31\ J/mol\text{-}K)/(298\ K) = 1.17\ kg/m^3 = 1.17 \times 10^{-3}\ g/cm^3$.

   Since the density is proportional to the pressure we can rewrite the barometric pressure formula.

$$\rho = \rho_0 \exp\left\{-\frac{Mgh}{RT}\right\}$$

and then solve for h

$$h = -\frac{RT}{Mg}\ln\left(\frac{\rho}{\rho_0}\right) = \frac{RT}{Mg}\ln\left(\frac{\rho_0}{\rho}\right) = \frac{(8.31\ J/mol\text{–}K)(298\ K)}{(0.029\ kg/mol)(9.8\ m/s^2)}\ln\left(\frac{1.17}{0.161}\right)$$
$$= 17280\ m$$

   If you were very clever you might have realized that the ratio of the densities of He and the atomsphere at sea level is equal to the ratio of their molar masses M_0/M and so you have written:

$$h = \frac{RT}{Mg}\ln\left(\frac{M_0}{M}\right) = \frac{(8.31\ J/mol\text{–}K)(298\ K)}{(0.029\ kg/mol)(9.8\ m/s^2)}\ln\left(\frac{29}{4}\right)$$
$$= 17280\ m$$

   to get the same answer.

4. We calculated the root-mean-square speed of O_2 using the kinetic theory of gases.
   A. What is the mean free path of O_2 at room temperature and at sea level.
   Solution: We need the collision cross section, σ, of O_2. This is given in Table 1.3 on page 26 of Atkins, $\sigma = 0.40\ nm^2$. The mean free path is:

$$\lambda = \frac{RT}{\sqrt{2}N_A\sigma P} = \frac{(8.31 J/mol\text{–}K)(298\ K)}{(1.41)(6.023\times 10^{23}\ /\ mol)(0.4\times 10^{-18} m^2)(10^5 Pa)}$$
$$= 7.3\times 10^{-8}\ m = 73\ nm$$

   B. Calculate the collision frequency of O_2 at room temperature and at sea level.
   Solution: The collision frequency is:

$$z = \frac{\sqrt{2}N_A\sigma\sqrt{<u^2>}P}{RT} = \frac{\sqrt{<u^2>}}{\lambda}$$

$$\sqrt{<u^2>} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3(8.31 J/mol\text{–}K)(298\ K)}{0.032\ kg\ /\ mol}} = 481.8\ m/s$$

$$z = \frac{481.8\ m\ /\ s}{7.3\times 10^{-8}\ m} = 6.6\times 10^{9}\ s^{-1}$$