Homework #1 Solutions

1. Calculate a realistic estimate for the temperature at the surface of Mars using the black body radiation formula. Use the following facts.
   Radius of Mars: 2110 miles (convert to meters).
   Distance of Mars from the sun: 142,000,000 miles (convert to meters).

   Solution: R_{mars} = 2110 miles x 1.62 km/mile x 1000 meters/m = 3.42 x 10^6 m
   R_{to_sun} = 1.42 x 10^8 miles x 1.62 km/mile x 1000 meters/m = 2.3 x 10^{11} m
   Given the radiant power of the sun at its surface is 3.2 x 10^{26} W as we calculated in class, the insolation on Mars is given by this power divided by the area of a sphere at the radius equal to the distance from Mars to the sun.
   The area is $A = 4\pi R_{to\_sun}^2 = 4(3.14159)(2.3 \times 10^{11}\ m)^2 = 6.65 \times 10^{23}\ m^2$.
   The insolation is $I = 3.2 \times 10^{26}\ W/6.65 \times 10^{23}\ m^2 = 481\ W/m^2$.
   It is about half as much flux as we receive here on earth.

   Now we calculate the total power that Mars absorbs. For this we need the cross sectional area of Mars.
   R_{mars} = 2100 miles = 3420 km = 3,420,000 m
   $A_{mars\_cs} = \pi R_{mars}^2 = 3.14159(3.42 \times 10^6\ m) = 3.67 \times 10^{13}\ m^2$.
   Thus the total power is:
   $P_{abs} = IA_{mars\_cs} = (481\ W/m^2)(3.67 \times 10^{13}\ m^2) = 1.76 \times 10^{16}\ W$.
   The corresponding value for the earth is 6 x 10^{17} W or about 40 times larger.
   Now, we must reason that if Mars is in equilibrium the power emitted as black body radiation must equal the power absorbed.
   $P_{emit} = P_{abs}$
   And
   $P_{emit} = \sigma T_{mars}^4 A_{mars} = P_{abs}$
   $T_{mars} = (P_{abs} / A_{mars}/\sigma)^{1/4}$
   $= (1.76 \times 10^{16}\ W/1.47 \times 10^{14}\ m^2/5.67 \times 10^{-8}\ kg\ s^{-3}\ K^{-4})^{1/4}$
   = 214 K.

2. What wavelength is the peak of the black body emission from Mars?

   Solution:
   $\lambda_{max}T = 2.88 \times 10^6$ nm-K
   $\lambda_{max} = 2.88 \times 10^6$ nm-K/T = 2.88 x 10^6 nm-K/120 K = 24000 nm = 24 μ

3. Which quantum numbers give rise to the transitions in the Balmer series for hydrogen as presented in lecture. Show your work.

   Solution: Using MAPLE we can see that the Balmer series is n = 2 to n = 3, 4, 5… etc.
   > evalf(1e7/109676/(1/4-1/9)); [evalf means evaluate floating point number]
   656.4790838

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> evalf(1e7/109676/(1/4-1/16));
                        486.2808028
> evalf(1e7/109676/(1/4-1/25));
                        434.1792882
> evalf(1e7/109676/(1/4-1/36));
                        410.2994274
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4. What is the DeBroglie wavelength of a 500 eV electron in an electron microscope?

   Solution:
   1 eV = 1.62×10^{-19} J
   So E = 500 eV = 8.1×10^{-17} J

   Use the electron mass to calculate the momentum.
   $E = p^2/2m$ so $p = \text{sqrt}(2mE) = \text{sqrt}(2 * 9.1 \times 10^{-31}\text{kg} * 8.1 \times 10^{-17}\ \text{J})$
   $p = 1.21 \times 10^{-23}$ kg m/s
   $\lambda = h/p = (6.626 \times 10^{-34}\ \text{Js})/(1.21 \times 10^{-23}\ \text{kg m/s})$
   $\lambda = 5.47 \times 10^{-11}$ m = 5.47×10^{-2} nm = 0.0547 nm.

   It is a mistake to use $E = h\nu$. This applies only to electromagnetic waves or their associated particles known as photons.
   $E = hc/\lambda$ or $\lambda = hc/E = (6.626 \times 10^{-34}\ \text{Js})(2.99 \times 10^{8}\ \text{m/s})/ (8.1 \times 10^{-17}\ \text{J})$
   $\lambda = 2.44 \times 10^{-9}$ m = 2.44 nm which is about 5 times too big!

5. What is the DeBroglie wavelength of a basketball that weighs 2 kg. if it is dropped from a height of 1 meter (you may neglect friction)?

   Solution:
   $mgh = 1/2mv^2 = p^2/2m$
   $p^2 = 2m^2gh$
   $p^2 = m\ \text{sqrt}(2gh) = 2\ \text{kg sqrt}(2*9.8\ \text{m/s}*1\ \text{m}) = 8.85$ kg m/s
   $\lambda = (6.626 \times 10^{-34}\ \text{Js})/(\ 8.85\ \text{kg m/s}) \sim 10^{-34}\ \text{m} \sim 10^{-25}$ nm,
   which is really hard to measure!