Homework #3 Due: Sept. 17

Name ________________________ Chemistry 331

1. A hiker caught in a rainstorm absorbs 1 liter of water in his/her clothing. It is windy so that this volume is quickly evaporated at 20^{o}C (the heat of vaporization of water is 2447 kJ/kg at this temperature).

   a. If the hiker is the "system" calculate q, the heat transferred when 1 L evaporates. (10 points)

   Solution: $q = -m\ \Delta_{vap}H = \rho\ V\ \Delta_{vap}H = -\ (1\ kg/L)(1\ L)(2447\ kJ/kg) = -2447\ kJ$

b. If all this heat were removed from the hiker what drop in body temperature would the hiker experience (ignore the metabolism of the hiker)? The hiker weighs 60 kg and has a specific heat capacity equal to that of water (4.18 kJ/kg-K). (15 points)

Solution:

$q = -m\ C_P\ \Delta T \rightarrow \Delta T = q/(m\ C_P) = -2447\ kJ/-(60\ kg)(4.18\ kJ/kg\text{-}K) = 9.75\ K$ or $9.75\ ^{o}C$

c. How many grams of sucrose would the hiker have to metabolize (quickly) to replace the heat of evaporating one liter of water to maintain his/her original body temperature? You can use the heat of reaction for the combustion of sucrose at 25^{o}C. The heat of formation of sucrose is 2222 kJ/mol. The reaction is (15 points):

Sucrose (s) + oxygen (g) $\rightarrow$ carbon dioxide (g) + water (l)

Solution: The reaction is $C_{12}H_{22}O_{11} + O_2 \rightarrow 12\ CO_2 + 11\ H_2O$

$\Delta_rH = 11\ \Delta_fH\ (H_2O) + 12\ \Delta_fH\ (CO_2) - \Delta_fH\ (C_{12}H_{22}O_{11})$

$= 11(-285.8\ kJ/mol) + 12(-393.5\ kJ/mol) - (-2222\ kJ/mol) = -5640\ kJ/mol$

grams of sucrose $= q\ M_{sucrose}/\Delta_rH = -2447\ kJ/mol(342\ g/mol)/(-5640\ kJ/mol) = 148$ grams.

2. a. An engineer is designing a solar heating unit for a house. A bed of granite rocks with a surface area of 100 m^2 and volume of 10 m^3 will be heated by direct sunlight. The specific heat of the rocks is 4 $Jg^{-1}K^{-1}$ and the average density is 4000 kg/m^3. Assume that the effective radiant flux (power per unit area) of the sun is 1 kW/m^2 and that there are six hours of effective heating per day. Assuming a morning temperature of 298 K, perfect heat transfer, and no losses, what final temperature will be achieved by the rocks at the end of the day? (10 points)

Solution: The total energy of sunlight that impinges on the rocks is:

Energy = Flux(Power per unit area) x Time x Area = $(1\ kW/m^2)(21600\ s)(100\ m^2)$

$= 2,160,000\ kJ.$

$q = m\ C_P\ \Delta T \rightarrow \Delta T = q/m\ C_P = q/\rho\ V\ C_P = 2,160,000\ kJ/(4000\ kg/m^3)(10\ m^3)(4\ kJ/kg/K)$

$= 13.5\ K$

The final temperature is 298 + 13.5 = 311.5 K.

b. Assuming perfect heat exchange between the rocks and pumped air, what volume of air (C_P = 29.0 $Jmol^{-1}K^{-1}$) can be heated by 10 K? (15 points)

Solution:
$n_{air}\ C_{P,air}\ \Delta T = q \rightarrow n_{air} = q / (C_{P,air}\ \Delta T)$ = 2,160,000,000 J/(29 J/mol-K)(10 K) = 7.4×10^7 moles
$V = nRT/P = (7.4 \times 10^7$ moles)(0.08206)(308 K)/1 atm = 1.8×10^9 L

3. a. How much solar energy is required to heat 1 mole of circulation water from 300 K to 310 K? (5 points)

Solution: $q_P = \Delta H = n\ C_P\ \Delta T$ = (1 mol)(75.29 J/mol-K)(10 K) = 752 J.

b. A flow of air at 275 K enters a heat exchanger at 22.5 L/sec. The hot water (310 K) supplied from a flat plate solar panel enters the inner tubing of the heat exchanger at 0.018 L/sec. Assuming that the heat exchanger has no losses and that the water and air reach equilibrium as they exit calculate the temperature of the heated air (10 points).

Solution:
$n_{air} = PV/RT$ = (1 atm)(22.5 L)/(0.08206)(275 K) = 1.0 mole (per second)
$n_{water} = \rho V/M$ = (1000 g/L)(0.018 L)/18 g/mol = 1.0 mole (per second)
$n_{air}\ C_{P,air}(T_f - T_{air}) = n_{water}\ C_{P,water}(T_f - T_{water})$
$T_f = (n_{air}\ C_{P,air}\ T_{air} + n_{water}\ C_{P,water}\ T_{water}) / (n_{air}\ C_{P,air} + n_{water}\ C_{P,water})$
T_f = (29)(275)+(75.3)(310)/(29+75.3) = 300 K

c. Treating the heat exchange process as a constant pressure expansion of the atmosphere and compression of liquid water, calculate the volume change of 1 L of air (10 points).

Solution: $V_2/V_1 = T_2/T_1 \rightarrow V_2 = V_1\ (T_2/T_1)$ = (22.5 L)(300/275) = 24.5 L
$\Delta V = V_2 - V_1$ = 24.5 – 22.5 = 2.0 L

d. Calculate the entropy change in both the water and air separately and the total entropy change for the process. Be sure to include both temperature and volume changes for the air (10 points).

Solution: Since entropy is a state function we can calculate the entropy change for the volume increase and temperature rise separately.
$\Delta S_{air} = n\ R\ \ln(V_2/V_1) + n\ C_V\ \ln(T_2/T_1) = n\ C_P\ \ln(T_2/T_1)$
The last step is possible since as we saw in the previous step $V_2/V_1 = T_2/T_1$.
Therefore, $\Delta S_{air} = n\ C_P\ \ln(T_2/T_1)$ = (1 mol)(29 J/mol-K) ln(300/275) = 2.52 J/K
$\Delta S_{water} = n\ C_P\ \ln(T_2/T_1)$ = (1 mol)(75.3 J/mol-K) ln(300/310) = -2.46 J/K

$\Delta S_{total} = \Delta S_{air} + \Delta S_{water}$ = 2.52 J/K – 2.46 J/K = 0.06 J/K.