Introduce simplest idea. The integral is the inverse of the differential:

$$\int_a^b dx = x\Big|_a^b = b - a$$

Note that this is a definite integral. By this we mean that it has limits and it represents an area under a curve (or a line in this case). The indefinite integral is just a function.
Here it is:

$$\int dx = x$$

Give the class another example of a differential:

$$\int_a^b d(s^2) = s^2\Big|_a^b = b^2 - a^2$$

Note that the integrand does not have to be a differential.
For example,

$$\int_a^b s\,ds = \frac{s^2}{2}\Big|_a^b = \frac{b^2 - a^2}{2}$$

We can see this by examining the inverse operation (differentiation of s^2).

$$\frac{d}{ds}s^2 = 2s \;\Rightarrow \frac{1}{2}\frac{d}{ds}s^2 = s$$

Make the point that it does not matter what you call the variable of integration. It will be evaluated and will not appear in the final answer (for a definite integral).

The general case for differentiation of a polynomial is:

$$\frac{d}{dz}z^n = nz^{n-1}$$

The general case for integration of a polynomial is:

$$\int z^n = \frac{z^{n+1}}{n+1}$$

Now give a sample problem. Evaluate:

$$\int_1^2 a^3 = \frac{a^4}{4}\Big|_1^2 = 4 \;- \frac{1}{4} = 3\frac{3}{4}$$

Now turn to logarithms. The general case discussed in class is:

$$\int_a^b d(\ln\;(x)) = \ln(x)\Big|_a^b = \ln(b) - \ln(a) = \ln\left(\frac{b}{a}\right)$$

But note that

$$\frac{d}{dx}\ln(x) = \frac{1}{x}$$

Remind the class that this was demonstrated graphically in class (note section 1 did not see it because of the problem with the projector). But it is on the website.

Therefore,

$$\int_a^b \frac{dx}{x} = \ln(x)\Big|_a^b = \ln(b) - \ln(a) = \ln\left(\frac{b}{a}\right)$$

Give a sample problem. Evaluate:

$$\int_2^5 \frac{da}{a} = \ln\left(\frac{5}{2}\right) = 0.916$$

Now, we get tricky. We will introduce two integrals that they will see later in the course and let them solve them as math problems.

Suppose you have the following integral (i.e. give the left hand side and let them figure out the solution):

$$-nRT\int_{V_1}^{V_2} \frac{dV}{V} = -nRT\ln\left(\frac{V_2}{V_1}\right)$$

Once they have done this tell them that this is the reversible work of expansion and that it will be derived in lecture in about a week.

Now write the following on the board:

$$\int_{P_1}^{P_2} \frac{dP}{P} = -\frac{Mg}{RT}\int_{h_1}^{h_2} dh$$

This is actually the solution to a (simple) differential equation. Explain that they need to the integral on each side.

$$\ln\left(\frac{P_2}{P_1}\right) = -\frac{Mg}{RT}\left(h_2 - h_1\right)$$

Now, indicate that that this is called the “barometric pressure formula” and it gives the pressure dependence for the atmosphere for a height h2 above the earth’s surface (h1). Ask what the pressure is at sea level. The answer is that at h1 = 0 the pressure is P1 = 1 atm. If we substitute those values in and assume that P2 = P and h2 = h then we get

$$\ln\left(P\right) = -\frac{Mgh}{RT}$$

As a final example, we will take the Clausius-Clapeyron equation.

$$\int_{P_1}^{P_2} \frac{dP}{P} = \frac{\Delta H}{R}\int_{T_1}^{T_2} \frac{dT}{T^2}$$