Homework #1 Due: Sept. 8
Name _________________________ Chemistry 331

1. Calculate a realistic estimate for the temperature at the surface of Mercury using the black body radiation formula. Use the following facts.
   Radius of Mercury: 1510 miles (convert to meters).
   Distance of Mercury from the sun: 35,900,000 miles (convert to meters).
   Note: Mercury does not rotate on its axis. Please calculate the temperature on the sunny side! You may assume that the dark side is very cold (i.e. near T = 0 K).

   Solution: R_{merc} = 1510 miles x 1.62 km/mile x 1000 meters/m = 2.44×10^6 m
   $R_{to\_sun} = 3.59 \times 10^7$ miles x 1.62 km/mile x 1000 meters/m = 5.82×10^{10} m
   Given the radiant power of the sun at its surface is 3.2×10^{26} W as we calculated in class, the insolation on Mercury is given by this power divided by the area of a sphere at the radius equal to the distance from Mars to the sun.
   The area is $A = 4\pi R_{to\_sun}^2 = 4(3.14159)(5.82 \times 10^{10}\ m)^2 = 4.26 \times 10^{22}\ m^2$.
   The insolation is $I = 3.2 \times 10^{26}\ W/4.26 \times 10^{22}\ m^2 = 7520\ W/m^2$.
   It is about seven times as much flux as we receive here on earth.
   Now we calculate the total power that Mercury absorbs. For this we need the cross sectional area of Mercury.
   $A_{mars\_cs} = \pi R_{merc}^2 = 3.14159(2.44 \times 10^6\ m)^2 = 1.87 \times 10^{13}\ m^2$.
   Thus the total power is:
   $P_{abs} = IA_{mars\_cs} = (7520\ W/m^2)(1.87 \times 10^{13}\ m^2) = 1.40 \times 10^{17}\ W$.
   The corresponding value for the earth is 6×10^{17} W or about 4 times larger.
   Now, we must reason that if Mars is in equilibrium the power emitted as black body radiation must equal the power absorbed.
   $P_{emit} = P_{abs}$
   And
   $P_{emit} = \sigma T_{merc}^4 A_{merc} = P_{abs}$
   Since Mercury does not rotate the effective emitting area is the half sphere on the sunny side:
   $A_{merc} = 4\pi R_{merc}^2/2 = 2(3.14159)(2.44 \times 10^6\ m)^2 = 3.74 \times 10^{13}\ m^2$.
   $T_{mars} = (P_{abs} / A_{merc}/\sigma)^{1/4}$
   $= (1.4 \times 10^{17}\ W/3.74 \times 10^{13}\ m^2/5.67 \times 10^{-8}\ kg\ s^{-3}\ K^{-4})^{1/4}$
   $= 506$ K.

   Temperature = ______506 K____________________________.

2. What wavelength is the peak of the black body emission from Mercury?

   Solution:
   $\lambda_{max}T = 2.88 \times 10^6$ nm-K
   $\lambda_{max} = 2.88 \times 10^6$ nm-K/T $= 2.88 \times 10^6$ nm-K/506 K = 5690 nm = 5.69 μ

   Wavelength = _______5.69 microns________________________________

3. Which quantum numbers give rise to the transitions in the Lyman series for hydrogen. Show your work.
The Lyman series is: 121.5 nm, 102.5 nm, 97.2 nm, 94.9 nm, 93.7 nm, 93.0 nm, 92.6 nm etc.

Solution: Using MAPLE we can see that the Lyman series is
n = 1 to n = 2, 3, 4… etc.
> evalf(1e7/109676/(1-1/4)); [evalf means evaluate floating point number]
121.57
> evalf(1e7/109676/(1-1/9));
102.57
> evalf(1e7/109676/(1-1/16));
97.25
> evalf(1e7/109676/(1-1/25));
94.56
Use of MAPLE is not required, but it can help.

Answer: The quantum numbers for the transition are:
__2__ → ___1___ for 121.5 nm
__3__ → ___1__ for 102.5 nm
__4__ → ___1__ for 97.2 nm
__5__ → ___1__ for 94.9 nm
__6__ → ___1__ for 93.7 nm
__7__ → ___1__ for 93.0 nm
__8__ → ___1__ for 92.6 nm

4. What is the DeBroglie wavenumber (in cm^{-1}) of a 2.5 keV electron in an electron microscope?

Solution:
1 eV = 1.62×10^{-19} J
So E = 2500 eV = 4.05×10^{-16} J

Use the electron mass to calculate the momentum.
$E = p^2/2m$ so p = sqrt(2mE) = sqrt(2 * 9.1×10^{-31}kg * 4.05×10^{-16} J)
p = 2.71×10^{-23} kg m/s
$\lambda = h/p = (6.626 \times 10^{-34}$ Js$)/(2.71 \times 10^{-23}$ kg m/s$)$
$\lambda = 2.44 \times 10^{-11}$ m = 2.44×10^{-2} nm = 0.0244 nm.
$\bar{\nu} = 10^7/0.0244$ nm = 4.1×10^8 cm^{-1}

It is a mistake to use E = hν. This applies only to electromagnetic waves or their associated particles known as photons.
$E = hc/\lambda$ or $\lambda = hc/E = (6.626 \times 10^{-34}$ Js$)(2.99 \times 10^8$ m/s$)/(4.05 \times 10^{-16}$ J$)$
$\lambda = 4.9 \times 10^{-10}$ m = 0.49 nm which is about 20 times too big!

DeBroglie wavenumber ________ 4.1×10^{8} cm^{-1} __________ **(cm^{-1})**

5. What is the DeBroglie wavelength of a truck that weighs 16 tons traveling at 60 miles per hour (you may neglect friction)?

   Solution:
   m = 1 ton = 2000 lbs = 908 kg
   v = 60 miles/hour = 97,200 m/3600 sec = 2.7 m/s
   mv = 908 kg(2.7 m/s) = 2452 kgm/s
   $\lambda = (6.626 \times 10^{-34}$ Js)/(2452 kg m/s) ~ 2.7×10^{-37} m ~ 2.7×10^{-28} nm,
   which is really hard to measure!

   DeBroglie wavelength __________ 2.7×10^{-28} nm _________

6. Solve the following integrals. Provide a sketch of each of them.

$$\int_0^{\infty} e^{-kx}dx = \frac{-1}{k}\int_0^{-\infty} e^{u}du = \frac{-1}{k}e^{u}\Big|_0^{-\infty} = \frac{-1}{k}(0\ -1) = \frac{1}{k}$$

$$\int_1^{10} \frac{dx}{x} = \ln(x)\Big|_1^{10} = \ln(10) - \ln(1) = 2.3025$$

$$\int_0^{10} xdx = \frac{x^2}{2}\Big|_0^{10} = \frac{10^2}{2} = 50$$

Note: You could add a constant of integration to each of these. I have omitted this.